Question 1

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A polynomial is shown.

\[ y = 4x^3 - 12x^2 - 4x + 12 \]

Use the Add Point tool to show the locations of all zeros of this polynomial.

Scoring Rubric:
3 points
For this item, the response correctly
• identifies all 3 zeros.

2 points
For this item, the response correctly
• identifies 2 of 3 zeros.
For this item, the response correctly
• identifies 1 of 3 zeros.

**Sample Correct Answer:**

A polynomial is shown.

\[ y = 4x^3 - 12x^2 - 4x + 12 \]

Use the Add Point tool to show the locations of all zeros of this polynomial.

**Explanation of Correct Answer:**

The equation \( y = 4x^3 - 12x^2 - 4x + 12 \) can be factored as shown.

\[
\begin{align*}
y &= 4x^3 - 12x^2 - 4x + 12 \\
y &= 4(x^3 - 3x^2 - x + 3) \\
y &= 4(x^2(x - 3) - (x - 3)) \\
y &= 4(x^2-1)(x - 3) \\
y &= 4(x + 1)(x - 1)(x - 3) \\
0 &= 4(x + 1)(x - 1)(x - 3)
\end{align*}
\]

From these factors, it is apparent that the zeros of the polynomial are \( x = -1 \), \( x = 1 \), and \( x = 3 \).
Question 2

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** A-REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

An equation is shown.

\[
\frac{1}{(x + 13)^2} = 10
\]

What is the solution to the equation?

---

**Scoring Rubric:**

1 point
For this item, the response correctly
• identifies the value.
Sample Correct Answer:

An equation is shown.

\[
\frac{1}{(x + 13)^2} = 10
\]

What is the solution to the equation?

87

Explanation of Correct Answer:

The equation is solved using the steps shown.

\[
(x + 13)^\frac{1}{2} = 10
\]

\[
[(x + 13)^\frac{1}{2}]^2 = 10^2
\]

\[
x + 13 = 100
\]

\[
x = 87
\]

Then, since substituting this value into the original equation results in a true statement, the solution for \( x \) is 87.
Question 3

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-REI.4b: Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a \pm bi \) for real numbers \( a \) and \( b \).

A quadratic equation is shown.

\[ 4(x + 7)^2 = 11 \]

Solve the equation and create one possible solution in radical form.

Scoring Rubric:

1 point
For this item, the response correctly
• identifies one equivalent solution.
Sample Correct Answer:

A quadratic equation is shown.

\[ 4(x + 7)^2 = 11 \]

Solve the equation and create one possible solution in radical form.

\[ x = -7 + \frac{\sqrt{11}}{2} \]
Explanation of Correct Answer:

To solve the equation, divide both sides by 4, take the square root of both sides, and then subtract 7 from both sides as shown.

\[ 4(x + 7)^2 = 11 \]
\[ (x + 7)^2 = \frac{11}{4} \]
\[ \sqrt{(x + 7)^2} = \pm \sqrt{\frac{11}{4}} \]
\[ x + 7 = \pm \frac{\sqrt{11}}{2} \]
\[ x = -7 \pm \frac{\sqrt{11}}{2} \]

Thus, one solution to the equation is \( x = -7 + \frac{\sqrt{11}}{2} \), and the other is \( x = -7 - \frac{\sqrt{11}}{2} \). Other equivalent expressions are also acceptable.

Sequence of keypad clicks to enter the answer.

\[ x, =, -, 7, +, \sqrt{}, 11, \text{click in the denominator}, 2 \]
Question 4

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** F-IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

The path of a ball follows a function, as shown.

\[ h(t) = -4.9t^2 + 4 \]

Move the parabola shown to graph the function \( h(t) \).

---

**Scoring Rubric:**

1 point
For this item, the response correctly
- places the graph.
Sample Correct Answer:

The path of a ball follows a function, as shown.

\[ h(t) = -4.9t^2 + 4 \]

Move the parabola shown to graph the function \( h(t) \).

Explanation of Correct Answer:

The vertex of a quadratic function \( f(x) \) occurs at the point where \( x = -\frac{b}{2a} \). Thus, for \( h(t) \), the value of \( t \) at the vertex is \( t = -\frac{0}{2(-4.9)} = 0 \). Then, since \( h(0) = 4 \), the correct graph is given by placing the vertex of the given parabola at (0, 4).
Question 5

Reporting Category: Modeling & Problem Solving

Common Core Standard: A-REI.4a: Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form.

Answer Key: B

Kelly is completing the square to solve the equation \(2x^2 - 32x = 10\).

Which equation could be the result of completing the square?

A. \((x - 8)^2 = 5\)

*This answer is not correct. The value of \(8^2\) was only added to the left side of the equation.*

B. \((x - 8)^2 = 69\)

*This answer is correct. This equation is in completed square form.*

C. \(2(x - 8)^2 = 10\)

*This answer is not correct. The value of \(2(8^2)\) was only added to the left side of the equation.*

D. \(2(x - 8)^2 = 74\)

*This answer is not correct. The value of \(8^2\) was added to the right side of the equation instead of the value of \(2(8^2)\).*
Question 6

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** F-IF.7b: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

The table of values represents an absolute value function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

A. Use the Add Point tool to plot the minimum of this function.

B. Drag numbers into the boxes and an operation symbol into the circle to create the equation of this function.
Scoring Rubric:

2 points
For this item, the response correctly:
- identifies the correct point
  AND
- identifies the correct equation.

1 point
For this item, the response correctly:
- identifies either the correct point or the correct equation.

Sample Correct Answer:

The table of values represents an absolute value function.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

A. Use the Add Point tool to plot the minimum of this function.

B. Drag numbers into the boxes and an operation symbol into the circle to create the equation of this function.

\[ y = \boxed{2}x \quad \boxed{-3} \]
Explanation of Correct Answer:

For part A, an absolute value function that opens up has a minimum at its vertex. From the table, the x-coordinate of the vertex must be halfway between the two points with the same y-coordinate, (1,1) and (2,1). Thus, the x-coordinate is $\frac{3}{2}$. Then since the y-value to the left of $x = \frac{3}{2}$ decreases by 2 each time the x-value increases by 1, the y-value at $x = \frac{3}{2}$ for the function is one less than the y-value at $x = 1$. Thus, the vertex is at $\left(\frac{3}{2}, 0\right)$.

For part B, the equation requires the coefficient of the x term and the constant value to be identified. Since the y-value to the right of $x = \frac{3}{2}$ increases by 2 each time the x-value increases by 1, the function has a slope of 2 to the right of $x = \frac{3}{2}$. Also since the point (1,1) is on the graph to the right of $x = \frac{3}{2}$, the constant term should be 3. Thus, the function is $y = |2x - 3|$.
Algebra II End of Course Exam Answer Key
Segment II

Scientific/Graphing/Regression Calculator Allowed
Question 7

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: N-RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

An expression in exponential form is shown.

\[
\frac{1}{x^{3/2}}
\]

Create the equivalent radical form of the expression.

Scoring Rubric:

1 point
For this item, the response correctly
• identifies the equivalent expression in radical form.
Sample Correct Answer:

An expression in exponential form is shown.

\[ \frac{1}{x^3} \]

Create the equivalent radical form of the expression.

\[ \frac{3}{\sqrt{x}} \]

Explanation of Correct Answer:

The radical form \( \frac{3}{\sqrt{x}} \) is equivalent to \( x^{\frac{1}{3}} \). The multiplication rule for exponents is consistent in radical and exponential form, allowing equations such as \( (\frac{3}{\sqrt{x}})^3 = \left(x^{\frac{1}{3}}\right)^3 = x \) to be true.

Sequence of keypad clicks to enter the answer.

\[ \sqrt[3]{x}, 3, \Rightarrow, x \]
Question 8

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: N-RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

An expression is shown.

\[ \left( \frac{64y^9}{x^{-3}} \right)^{\frac{1}{3}} \]

Create an equivalent expression.

Use only positive integer exponents in your answer.

Scoring Rubric:

1 point
For this item, the response correctly
- identifies an equivalent simplified form of the expression.
Sample Correct Answer:

An expression is shown.

\[
\left( \frac{64y^9}{x^{-3}} \right)^{\frac{1}{3}}
\]

Create an equivalent expression.

Use only positive integer exponents in your answer.

\[
4xy^3
\]

Explanation of Correct Answer:

The steps to simplify the expression are shown.

\[
\left( \frac{64y^9}{x^{-3}} \right)^{\frac{1}{3}} = (64y^9x^3)^{\frac{1}{3}}
\]

\[
= (64)^{\frac{1}{3}}(y^9)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}
\]

\[
= 4y^3x
\]

\[
= 4xy^3
\]

Sequence of keypad clicks to enter the answer.

4, x, y, □□, 3
Question 9

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Answer Key: B

Evan buys food for his dog. The table shows four options.

<table>
<thead>
<tr>
<th>Option</th>
<th>Price per Bag</th>
<th>Amount of Food (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$24</td>
<td>15.4</td>
</tr>
<tr>
<td>2</td>
<td>$30</td>
<td>20.8</td>
</tr>
<tr>
<td>3</td>
<td>$39</td>
<td>25.4</td>
</tr>
<tr>
<td>4</td>
<td>$25</td>
<td>16.5</td>
</tr>
</tbody>
</table>

Evan's dog eats 1 cup of food per meal. Each cup contains $\frac{1}{3}$ pound of food.

What is the lowest price per cup Evan could pay, rounded to the nearest cent?

A. $0.46

*This answer is not correct. The student may have converted pounds to cups for the first option.*

B. $0.48

*This answer is correct. The student identified the correct cost of the least expensive food. Since each cup = $\frac{1}{3}$ pound, multiply each amount of food option by 3 to convert from pounds to cups. This will result in 46.2, 62.4, 76.2, and 49.5. Then divide each price by its cups to get the price per cup. (0.52, 0.48, 0.51, 0.51)*
C. $0.51

This answer is not correct. The student may have identified the price per cup of the third bag of food.

D. $0.52

This answer is not correct. The student may have identified the price per cup of the first bag and thought that it was the lowest cost per cup due to its price per bag.
Question 10

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** N-CN.1: Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.

A number is shown.

$$\sqrt{-25}$$

Show the value of this number in $a + bi$ form.

**Scoring Rubric:**

1 point
For this item, the response correctly
- identifies an equivalent value.
Sample Correct Answer:

A number is shown.

\[ \sqrt{-25} \]

Show the value of this number in \( a + bi \) form.

\[ 0 + 5i \]

Explanation of Correct Answer:

The steps to write the expression in \( a + bi \) form are shown.

\[
\sqrt{-25} = \sqrt{25} \sqrt{-1} \\
= \sqrt{5^2} \sqrt{-1} \\
= 5i \\
= 0 + 5i
\]
Question 11

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: N-CN.2: Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Two equations are shown.

\[ a_1 = \frac{3}{4} + \frac{1}{2}i \]
\[ a_2 = \frac{1}{4} + \frac{1}{6}i \]

What is the value of $a_1 - a_2 + \frac{3}{7}$ in $a + bi$ form?

Scoring Rubric:

1 point
For this item, the response correctly
• identifies an equivalent value.
Sample Correct Answer:

Two equations are shown.

\[ a_1 = \frac{3}{4} + \frac{1}{2}i \]
\[ a_2 = \frac{1}{4} + \frac{1}{6}i \]

What is the value of \( a_1 - a_2 + \frac{3}{7} \) in \( a+bi \) form?

\[ \frac{13}{14} + \frac{1}{3}i \]
Explanation of Correct Answer:

The calculations used to find $a_1 - a_2 + \frac{3}{7}$ are shown below.

$$a_1 - a_2 + \frac{3}{7} = \left(\frac{3}{4} + \frac{1}{2}i\right) - \left(\frac{1}{4} + \frac{1}{6}i\right) + \frac{3}{7}$$

$$= \frac{3}{4} + \frac{1}{2}i - \frac{1}{4} - \frac{1}{6}i + \frac{3}{7}$$

$$= \frac{3}{4} - \frac{1}{4} + \frac{3}{7} + \frac{1}{2}i - \frac{1}{6}i$$

$$= \frac{21}{28} - \frac{7}{28} + \frac{12}{28} + \frac{3}{6}i - \frac{1}{6}i$$

$$= \frac{26}{28} + \frac{2}{6}i$$

$$= \frac{13}{14} + \frac{1}{3}i$$

Sequence of keypad clicks to enter the answer.

- , 13, click in the denominator, 14, ➔, +, , 1, click in the denominator, 3, ➔, i
Question 12

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: N-CN.7: Solve quadratic equations with real coefficients that have complex solutions.

An expression is shown.

\[ 5x^2 + 2x + 1 = 0 \]

What is one possible solution to this quadratic equation?

Scoring Rubric:

1 point
For this item, the response correctly
• identifies one of the two solutions.
Sample Correct Answer:

An expression is shown.

\[ 5x^2 + 2x + 1 = 0 \]

What is one possible solution to this quadratic equation?

\[ -0.2 + 0.4i \]
Explanation of Correct Answer:

The equation $5x^2 + 2x + 1 = 0$ can be solved using the quadratic formula as shown.

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
= \frac{-(2) \pm \sqrt{(2)^2 - 4(5)(1)}}{2(5)}
= \frac{-2 \pm \sqrt{-16}}{10}
= \frac{-2 \pm 4i}{10}
= -0.2 \pm 0.4i
$$

Therefore, one solution to the equation is $-0.2 + 0.4i$, and the other solution is $-0.2 - 0.4i$. 
Question 13

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-SSE.2: Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Create an expression that represents the complete factorization of $2x^2 + 16x + 32$.

Scoring Rubric:

1 point
For this item, the response correctly
• identifies the factored expression.
Sample Correct Answer:

Create an expression that represents the complete factorization of $2x^2 + 16x + 32$.

$$2(x+4)^2$$

Explanation of Correct Answer:

The expression can be factored by removing the common factor and using the perfect-square trinomial formula as shown.

$$2x^2 + 16x + 32 = 2(x^2 + 8x + 16)$$
$$= 2(x^2 + (4 + 4)x + 4^2)$$
$$= 2(x + 4)(x + 4)$$
$$= 2(x + 4)^2$$

Sequence of keypad clicks to enter the answer.

2, (), x, +, 4, ➔, □, 2
Question 14

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-SSE.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

An equation is shown.

\[3x^2 + 14x = 5\]

Re-create the equation in an equivalent factored form to reveal the zeros.

Scoring Rubric:

1 point
For this item, the response correctly
• identifies the equation.
Sample Correct Answer:

An equation is shown.

\[ 3x^2 + 14x = 5 \]

Re-create the equation in an equivalent factored form to reveal the zeros.

\[
(3x - 1)(x + 5) = 0
\]

Explanation of Correct Answer:

The equation can be factored using the steps shown.

\[
3x^2 + 14x = 5 \\
3x^2 + 14x - 5 = 0 \\
3x^2 + 15x - x - 5 = 0 \\
3x^2 + (15 - 1)x - 5 = 0 \\
(3 \cdot 1)x^2 + [(3 \cdot 5) + (1 \cdot -1)]x + (5 \cdot -1) = 0 \\
(3x - 1)(x + 5) = 0
\]

Sequence of keypad clicks to enter the answer.

\[ (, 3, x, -, 1, \rightarrow, (), x, +, 5, \rightarrow, =, 0 \]
Question 15

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Create an expression that is equivalent to \((4x^2 + 7x - 9) - (-2x^2 - 4x + 6)\).

Scoring Rubric:

1 point
For this item, the response correctly
• identifies the expression.
Sample Correct Answer:

Create an expression that is equivalent to \((4x^2 + 7x - 9) - (-2x^2 - 4x + 6)\).

\[
6x^2 + 11x - 15
\]

Explanation of Correct Answer:

The difference of the two polynomials can be simplified as shown.

\[
(4x^2 + 7x - 9) - (-2x^2 - 4x + 6) \\
= 4x^2 + 7x - 9 - (-2x^2) - (-4x) - 6 \\
= 4x^2 + 7x - 9 + 2x^2 + 4x - 6 \\
= 4x^2 + 2x^2 + 7x + 4x - 9 - 6 \\
= 6x^2 + 11x - 15
\]

Sequence of keypad clicks to enter the answer.

6, x, □, 2, ➔, +, 11, x, −, 15
Question 16

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** A-APR.4: Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity 

\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]

can be used to generate Pythagorean triples.

A square has side length \(x\).

A new square is created by subtracting \(y\) from each side of the original square.

Create an expression for the area of the new square in expanded form.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>&lt;</td>
<td>≤</td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>(\div)</td>
<td>( )</td>
<td>(\sqrt{\phantom{0}})</td>
</tr>
</tbody>
</table>

**Scoring Rubric:**

1 point

For this item, the response correctly
- identifies an expression in expanded form.
Sample Correct Answer:

A square has side length $x$.

A new square is created by subtracting $y$ from each side of the original square.

Create an expression for the area of the new square in expanded form.

$$x^2 - 2xy + y^2$$

Explanation of Correct Answer:

Since the original square has side length $x$ and the new square is created by subtracting $y$ from each side, the new square has side length $x - y$. Then the area of the new square is given by $(x - y)^2$. Thus, in expanded form, the area is $(x - y)^2 = (x - y)(x - y) = x^2 - 2xy + y^2$.

Sequence of keypad clicks to enter the answer.

$x, 2, \rightarrow, -, 2, x, y, +, y, 2$
Question 17

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** A-APR.5: Know and apply the Binomial Theorem for the expansion of \((x + y)^n\) in powers of \(x\) and \(y\) for a positive integer \(n\), where \(x\) and \(y\) are any numbers, with coefficients determined for example by Pascal’s Triangle.

What is the coefficient of the \(x^4\) term in the binomial expansion of \((x + 1)^6\)?

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>-</td>
</tr>
</tbody>
</table>

**Scoring Rubric:**

1 point
For this item, the response correctly
- identifies an equivalent value.
Sample Correct Answer:

What is the coefficient of the \(x^4\) term in the binomial expansion of \((x + 1)^6\)?

15

Explanation of Correct Answer:

The Binomial Theorem states that the coefficient \(a\) in the \(x^b y^c\) term of the expansion of \((x + y)^n\) is given by \(\binom{n}{b}\) or \(\binom{n}{c}\) (which have the same value). Since the \(x^4\) term is the same as the \(x^4(1)^2\) term in the expansion of \((x + 1)^6\), this coefficient is given by \(\binom{6}{4} = \binom{6}{2}\). This binomial coefficient is computed as shown.

\[
\binom{6}{4} = \frac{6!}{4!(6 - 4)!}
= \frac{6!}{4!2!}
= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}
= 15
\]
Question 18

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-APR.6: Rewrite simple rational expressions in different forms; write \( \frac{a(x)}{b(x)} \) in the form \( q(x) + \frac{r(x)}{b(x)} \), where \( a(x) \), \( b(x) \), \( q(x) \), and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system.

A rational expression is shown.

\[
\frac{x^4 - 1}{x + 1}
\]

What is the quotient?

Scoring Rubric:

1 point
For this item, the response correctly
• identifies the quotient.
Sample Correct Answer:

A rational expression is shown.

\[ \frac{x^4 - 1}{x + 1} \]

What is the quotient?

\[ x^3 - x^2 + x - 1 \]

Explanation of Correct Answer:

The quotient can be found using polynomial long division as shown.

\[
\begin{align*}
&x^3 - x^2 + x - 1 \\
&(x + 1) | x^4 + 0x^3 + 0x^2 + 0x - 1 \\
&- (x^4 + x^3) \\
&- x^3 + 0x^2 \\
&- (x^3 - x^2) \\
&x^2 + 0x \\
&- (x^2 + x) \\
&-x - 1 \\
&- (-x - 1) \\
&0
\end{align*}
\]

Since there is no remainder, the quotient written in \( q(x) + \frac{r(x)}{b(x)} \) form is \( x^3 - x^2 + x - 1 \).

Sequence of keypad clicks to enter the answer.

\[ x, \quad 3, \quad \rightarrow, \quad -, \quad x, \quad 2, \quad \rightarrow, \quad +, \quad x, \quad -, \quad 1 \]
Question 19

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: A-APR.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions

What is the difference of \( \frac{x}{x-5} \) and \( x^2 - 4x - 5 \)?

Scoring Rubric:

1 point
For this item, the response correctly
- identifies an expression equivalent to \( \frac{x-2}{x-5} \) or \( \frac{2-x}{x-5} \).
Sample Correct Answer:

What is the difference of $\frac{x}{x-5}$ and $\frac{2x+2}{x^2-4x-5}$?

$$\frac{x-2}{x-5}$$
Explanation of Correct Answer:

Since $x^2 - 4x - 5 = (x - 5)(x + 1)$, the least common denominator of $\frac{x}{x-5}$ and $\frac{2x+2}{x^2-4x-5}$ is $(x - 5)(x + 1)$. Then, write $\frac{x}{x-5}$ with the least common denominator by multiplying the numerator and denominator by $(x + 1)$ and complete the subtraction as shown.

\[
\frac{x}{x-5} - \frac{2x+2}{x^2-4x-5} = \frac{x(x+1)}{(x-5)(x+1)} - \frac{2(x+1)}{(x-5)(x+1)}
\]
\[
= \frac{x^2 + x - 2x - 2}{(x-5)(x+1)}
\]
\[
= \frac{x^2 - x - 2}{(x-5)(x+1)}
\]
\[
= \frac{(x - 2)(x + 1)}{(x - 5)(x + 1)}
\]
\[
= \frac{x - 2}{x - 5}
\]

Sequence of keypad clicks to enter the answer.

`[ ]`, `x`, `-`, `2`, click in the denominator, `x`, `-`, `5`
Question 20

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** A-CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$.

The original circumference of a tree, $c$, increased at a rate of $2\pi$ inches per year for $t$ years.

The final circumference is $141\pi$ inches.

A. Click all the equations that show the correct relationship.

B. Determine the original circumference, in terms of $\pi$, if $t = 3$. Drag numbers into the box to show your answer.

### Scoring Rubric:

**2 points**
For this item, the response correctly
- identifies both representative equations
AND
- finds the value at $t = 3$.

**1 point**
For this item, the response correctly
- answers either part A or part B.
Sample Correct Answer:

The original circumference of a tree, \( c \), increased at a rate of \( 2\pi \) inches per year for \( t \) years.

The final circumference is \( 141\pi \) inches.

A. Click all the equations that show the correct relationship.

B. Determine the original circumference, in terms of \( \pi \), if \( t = 3 \). Drag numbers into the box to show your answer.

Explanation of Correct Answer:

The first sentence could be translated into the expression \( c + 2\pi t \). The final circumference is \( 141\pi \), so an equation is \( c + 2\pi t = 141\pi \).

In part A, the two correct equations are \( c = 141\pi - 2\pi t \) and \( 2\pi = \frac{141\pi - c}{t} \).

In part B, substitute 3 for \( t \) in any of the correct equations, and calculate to find the answer, \( 135\pi \) inches.
Question 21

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** A-REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

The sum of two quadratic expressions is 3 times the difference of the expressions.

The first expression is shown.

\[ 4x^2 - 6x + 10 \]

Marcia's steps for finding the second expression are shown.

Let \( A = 4x^2 - 6x + 10 \)

Let \( B = \) second expression

\[
(A + B) = 3(A - B) \\
A + B = 3A - 3B \\
4B = 2A \\
4B = 2(4x^2 - 6x + 10) \\
4B = 8x^2 - 12x + 20
\]

Create an equation that shows the next step used to find \( B \).
Scoring Rubric:

1 point  
For this item, the response correctly
• identifies an equivalent equation.

Sample Correct Answer:

The sum of two quadratic expressions is 3 times the difference of the expressions.

The first expression is shown.

\[ 4x^2 - 6x + 10 \]

Marcy’s steps for finding the second expression are shown.

Let \( A = 4x^2 - 6x + 10 \)
Let \( B = \) second expression

\[ (A + B) = 3(A - B) \]

\[ A + B = 3A - 3B \]

\[ 4B = 2A \]

\[ 4B = 2(4x^2 - 6x + 10) \]

\[ 4B = 8x^2 - 12x + 20 \]

Create an equation that shows the next step used to find \( B \).

\[ B = 2x^2 - 3x + 5 \]
Explanation of Correct Answer:

In the next step, to find $B$, we divide both sides of the equation by 4 as shown.

\[
\begin{align*}
4B &= 8x^2 - 12x + 20 \\
\frac{4B}{4} &= \frac{8x^2 - 12x + 20}{4} \\
B &= 2x^2 - 3x + 5
\end{align*}
\]

Sequence of keypad clicks to enter the answer.

$B, =, 2, x, \square, 2, \rightarrow, -, 3, x, +, 5$
Question 22

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** A-REI.11: Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Two functions are shown.

\[ f(x) = 3x + 1 \]
\[ g(x) = -\frac{3}{2}x - 7 \]

What is the value of \( x \) when \( f(x) = g(x) \)?

**Scoring Rubric:**

1 point
For this item, the response correctly
- identifies an equivalent value.
Sample Correct Answer:

Two functions are shown.

\( f(x) = 3x + 1 \)

\( g(x) = \frac{3}{2}x - 7 \)

What is the value of \( x \) when \( f(x) = g(x) \)?

\[ \frac{-16}{9} \]

Explanation of Correct Answer:

To find the value of \( x \) where \( f(x) = g(x) \), set the functions equal to each other and solve as shown.

\[
\begin{align*}
  f(x) &= g(x) \\
  3x + 1 &= \frac{3}{2}x - 7 \\
  3x + \frac{3}{2}x &= -1 - 7 \\
  \frac{9}{2}x &= -8 \\
  x &= -\frac{16}{9}
\end{align*}
\]

Sequence of keypad clicks to enter the answer.

\(-, \div, 16, \text{click in the denominator}, 9\)
**Question 23**

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** F-IF.8b: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.02)^t \), \( y = (0.97)^t \), \( y = (1.01)^{12t} \), \( y = (1.2)^{\frac{t}{10}} \), and classify them as representing exponential growth or decay.

**Scoring Rubric:**

**1 point**
For this item, the response correctly
- identifies all the decay functions.

A set of exponential equations is shown.

Drag all of the functions that model exponential decay into the box.

**Exponential Decay**

\[
\begin{align*}
y &= (0.99)^x \\
y &= (0.86)^{\frac{x}{3}} \\
y &= (1.27)^{0.3x} \\
y &= (1.07)^x \\
y &= 0.99(0.12)^x \\
y &= 0.89(1.02)^x \\
y &= (1.01)^{0.1} 
\end{align*}
\]
Sample Correct Answer:

A set of exponential equations is shown.

Drag all of the functions that model exponential decay into the box.

Exponential Decay

\[
\begin{align*}
    y &= (0.99)^x \\
    y &= 0.99(0.12)^x \\
    y &= (0.86)^{\frac{x}{3}} \\
    y &= (1.27)^{0.3x} \\
    y &= (1.07)^x \\
    y &= 0.89(1.02)^y \\
    y &= (1.01)^{\frac{x}{3}}
\end{align*}
\]

Explanation of Correct Answer:

A function indicates exponential decay when its base is between 0 and 1. A function with a base greater than 1 will grow larger as \( x \), the exponent, increases. But a function with a base between 0 and 1 will grow smaller as \( x \) increases. For example, the following is true for \( y = (0.99)^x \).

When \( x = 1 \), \( y = 0.99 \).
When \( x = 2 \), \( y = 0.9801 \).
When \( x = 3 \), \( y = 0.970296 \).

Out of the set of seven functions, the three functions \( y = (0.99)^x \), \( y = 0.99(0.12)^x \), and \( y = (0.86)^{\frac{x}{3}} \) model exponential decay.
Question 24

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** F-IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

**Answer Key:** D

A graph is shown.

Which equation has the same minimum as this graph?

A. \( y = -x^2 - 9 \)

*This answer is not correct. This function has a maximum at \((0, -9)\).*
B. \( y = x^2 + 9 \)

*This answer is not correct. This function has a minimum at \((0, 9)\).*

C. \( y = -x^2 - 6x \)

*This answer is not correct. This function has a maximum at \((-3, 9)\).*

D. \( y = x^2 - 6x \)

*This answer is correct. This function has a minimum at \((3, -9)\). The minimum occurs where:*

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{6}{2(1)}
\]

\[
x = 3
\]

Substitute that back into the equation:

\[
y = (3)^2 - 6(3)
\]

\[
y = 9 - 18
\]

\[
y = -9
\]
**Question 25**

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** F-BF.3: Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Consider the quadratic function shown.

\[ f(x) = -2(x + 1)^2 \]

Move one parabola onto the coordinate grid to show the transformation \( f(x) + 3 \).

---

**Scoring Rubric:**

1 point
For this item, the response correctly
- places the vertex of the downward-facing parabola at \((-1, 3)\).
Sample Correct Answer:

Consider the quadratic function shown.

\[ f(x) = -2(x + 1)^2 \]

Move one parabola onto the coordinate grid to show the transformation \( f(x) + 3 \).

Explanation of Correct Answer:

The parabola should open down because the leading coefficient, \(-2\), is less than 0. The equation is in vertex form showing that its vertex is \((-1,0)\). The transformation represented by \( f(x) + 3 \) shifts \( f(x) \) vertically by 3 units. So, the vertex after the transformation should be \((-1, 3)\).
Question 26

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: F-BF.4a: Find inverse functions. a. Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse and write an expression for the inverse. For example, \( f(x) = 2x^3 \) or \( f(x) = \frac{x+1}{x-1} \) for \( x \neq 1 \).

What is the inverse function of \( g(x) = 3x^2 - 2, \ x \geq 0 \)?

Create a function \( f(x) \) that represents the inverse of \( g(x) \).

Scoring Rubric:

1 point
For this item, the response correctly
- identifies an equivalent function.
Sample Correct Answer:

What is the inverse function of \( g(x) = 3x^2 - 2, \ x \geq 0 \)?

Create a function \( f(x) \) that represents the inverse of \( g(x) \).

\[
f(x) = \sqrt{\frac{x+2}{3}}
\]

Explanation of Correct Answer:

To find the inverse of \( g(x) \), replace \( g(x) \) with \( x \) and \( x \) with \( y \) in the statement of \( g(x) \) and solve for \( y \) as shown.

\[
g(x) = 3x^2 - 2
\]
\[
x = 3y^2 - 2
\]
\[
x + 2 = 3y^2
\]
\[
\frac{x + 2}{3} = y^2
\]
\[
\pm \sqrt{\frac{x + 2}{3}} = y
\]

Then, since the value of \( x \) must be nonnegative in the original function \( g(x) \), the value of \( y \) in the new function must be nonnegative. Thus, the inverse of \( g(x) \) is \( f(x) = \sqrt{\frac{x+2}{3}} \).

Sequence of keypad clicks to enter the answer.

\[
f(x), =, \sqrt{, \ [}, \ - \ +, 2, \text{ click in the denominator, 3}
\]
Question 27

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: F-BF.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Answer Key: C

Zachary is studying the behavior of a group of cells in a lab. He starts with 500 cells and observes that the cell population triples every hour. The function shown models the number of cells after t hours.

\[ f(t) = 500(3)^t \]

Which equation can be used to determine how many hours it will take for the cell population to reach 50,000?

A. \( \log_{100} 3 = t \)

This answer is not correct. The student may have reversed the base and the argument.

B. \( \log_3 500 = t \)

This answer is not correct. The student may have known the base was 3 and inserted 500 incorrectly.

C. \( \log_3 100 = t \)

This answer is correct. The student correctly found the inverse of the exponential expression to solve for t.

\[ f(t) = 500(3)^t \]
\[ 50,000 = 500(3)^t \]
\[ 100 = 3^t \]
\[ \log_3 100 = \log_3 3^t \]
\[ \log_3 100 = t \]

D. \( \log_{500} 50,000 = t \)

This answer is not correct. The student may have multiplied 500 by 3 before taking the logarithm of both sides.
Question 28

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: F-LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Answer Key: C

Four functions are shown on the graph.

When $x > 100$, which function has the smallest $y$-values?

A. $f(x)$

This answer is not correct. The student may have thought a linear function would have the smallest values for $y$ when $x > 100$. 
B. \( g(x) \)

This answer is not correct. The student may have thought that a cubic function would have the smallest value of \( y \) when \( x > 100 \).

C. \( h(x) \)

This answer is correct. The student shows correct understanding of end behavior of functions.

D. \( j(x) \)

This answer is not correct. The student may have thought that a quadratic function with a maximum value would have the smallest value of \( y \) when \( x > 100 \).
Question 29

Reporting Category: Algebraic Concepts & Procedures

Common Core Standard: F-LE.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

An equation is shown.

$x = \log(20) + 2$

What is the exponential form of the equation?

Scoring Rubric:

1 point
For this item, the response correctly
• identifies an equivalent equation.
Sample Correct Answer:

An equation is shown.

\[ x = \log(20) + 2 \]

What is the exponential form of the equation?

\[ 10^{(x-2)} = 20 \]

Explanation of Correct Answer:

To write the exponential form of the equation, first subtract 2 from both sides of the equation to obtain \( x - 2 = \log(20) \). Then, recall that an equation of the form \( \log_b x = y \) is equivalent to \( b^y = x \). Thus, since the base in the equation \( x - 2 = \log(20) \) is 10, the equation is equivalent to \( 10^{(x-2)} = 20 \).

Sequence of keypad clicks to enter the answer.

\[ 10, 2, 3, x, 4, 5, 6, +, -, *, \div, 7, 8, 9, <, \le, =, \ge, >, 0, ., \frac{a}{b}, \sqrt{a}, \sqrt[3]{a}, ()\]
Question 30

Reporting Category: Modeling & Problem Solving

Common Core Standard: N-Q.2: Define appropriate quantities for the purpose of descriptive modeling.

Meg deposited $1000 into a bank account that pays 3% monthly interest. The formula for compound interest is shown.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

- \( P \) is the principal
- \( r \) is the rate
- \( n \) is the number of times compounded per year
- \( t \) is the time in years

Use the data in the graph to determine which units would be most appropriate to represent the information in the graph.

Drag a label to each box.

Scoring Rubric:

1 point
For this item, the response correctly
- identifies both labels.
Sample Correct Answer:

Meg deposited $1000 into a bank account that pays 3% monthly interest.

The formula for compound interest is shown.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

- \(P\) is the principal
- \(r\) is the rate
- \(n\) is the number of times compounded per year
- \(t\) is the time in years

Use the data in the graph to determine which units would be most appropriate to represent the information in the graph.

Drag a label to each box.

Explanation of Correct Answer:

In this case, the amount of money, \(A\), is dependent on the time, \(t\), so the unit dollars should be placed on the dependent, or vertical, axis. Then, the information in the problem states that the time is given in years, so the best label for the independent, or horizontal, axis is years.
Question 31

**Reporting Category:** Modeling & Problem Solving

**Common Core Standard:** A-CED.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Rebecca records the amount of money, \( f(x) \), in her bank account each month, \( x \), as shown in the table.

<table>
<thead>
<tr>
<th>Month ( x )</th>
<th>Amount of Money ( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$6</td>
</tr>
<tr>
<td>1</td>
<td>$12</td>
</tr>
<tr>
<td>2</td>
<td>$24</td>
</tr>
<tr>
<td>3</td>
<td>$48</td>
</tr>
<tr>
<td>4</td>
<td>$96</td>
</tr>
</tbody>
</table>

Create a function that models this relationship.

**Scoring Rubric:**

1 point
For this item, the response correctly
- identifies an equivalent function.
Sample Correct Answer:

Rebecca records the amount of money, \( f(x) \), in her bank account each month, \( x \), as shown in the table.

<table>
<thead>
<tr>
<th>Bank Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month ( (x) )</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Create a function that models this relationship.

\[ f(x) = 6(2)^x \]

**Explanation of Correct Answer:**

Use the Graphing Calculator tool. Select Regression. Enter the \( x \)-values in the \( x \) column. Enter the \( f(x) \) values in the Y1 column. Select Exponential. The equation displayed is \( Y1 = 6 \times (2)^{x} \). Thus, the correct function is \( f(x) = 6(2)^x \).

**Sequence of keypad clicks to enter the answer.**

\[ f(x), =, 6, (,), 2, \rightarrow, ^{3}, x \]
Question 32

Reporting Category: Modeling & Problem Solving

Common Core Standard: A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Mike's class is going on a field trip to the museum. The total cost of the trip includes the cost of the tickets to the museum and a parking fee. Each ticket costs $10, and the parking fee is $20.

A. Select all the equations that represent this situation, where $C$ is the total cost and $t$ is the number of tickets sold.

B. Use the Add Arrow tool to graph this situation.

A. Select Equations

\[
\begin{align*}
C &= 20t + 10 \\
-10t + C &= 20 \\
20 &= 10t - C
\end{align*}
\]

B. Graph the Situation

Scoring Rubric:

2 points
For this item, the response correctly

- identifies the representative equations
- graphs the equation.

1 point
For this item, the response correctly

- answers either part A or part B.
Sample Correct Answer:

Mike’s class is going on a field trip to the museum. The total cost of the trip includes the cost of the tickets to the museum and a parking fee. Each ticket costs $10, and the parking fee is $20.

A. Select all the equations that represent this situation, where $C$ is the total cost and $t$ is the number of tickets sold.

\[
C = 20t + 10 \quad C = 10t + 20 \\
-10t + C = 20 \quad 20 = 10t - C
\]

B. Use the Add Arrow tool to graph this situation.

Explanation of Correct Answer:

The situation can be modeled as a linear equation in slope-intercept form, where $C$ is the dependent variable and $t$ is the independent variable. Since $C$ increases by $10 for every ticket the class buys, 10 should be the coefficient of $t$ that gives the slope of the linear function. Since $C$ is $20 even when no tickets have been bought, at $t = 0$, 20 is the intercept. So the representative equation can be written as $C = 10t + 20$. Only one other equation in part A is equivalent: $-10t + C = 20$.

In part B, the line should start at $(0, 20)$, the intercept. It should then continue increasing at a rate of $10 per ticket, or with a slope of 10.
Question 33

**Reporting Category:** Modeling & Problem Solving

**Common Core Standard:** A-SSE.1a: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.

**Answer Key:** D

Four cattle ranches plan to increase the size of their herds. The expressions show the predicted herd size for each ranch after $n$ years.

**Ranch Data**

<table>
<thead>
<tr>
<th>Ranch Name</th>
<th>Herd Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar2</td>
<td>$100n + 2000$</td>
</tr>
<tr>
<td>FlyingT</td>
<td>$90n + 1000$</td>
</tr>
<tr>
<td>LazyJ</td>
<td>$100n + 3000$</td>
</tr>
<tr>
<td>TC</td>
<td>$120n + 1000$</td>
</tr>
</tbody>
</table>

Which ranch has the herd with the fastest growth rate?

A. Bar2

*This answer is not correct. The student may have selected the first name shown.*

B. FlyingT

*This answer is not correct. The student may have thought that the least slope would lead to the fastest growth.*

C. LazyJ

*This answer is not correct. The student may have chosen the ranch with the largest number shown.*

D. TC

*This answer is correct. The student correctly identified the equation with the greatest slope.*
Question 34

Reporting Category: Modeling & Problem Solving

Common Core Standard: F-IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Becky throws a ball into the air. The height of the ball in feet, \( f(t) \), after \( t \) seconds can be modeled by the function shown.

\[ f(t) = -16t^2 + 40t + 6 \]

What is the height of the ball, in feet, 2 seconds after Becky throws it?

Scoring Rubric:

1 point
For this item, the response correctly
• identifies the correct height for the ball after 2 seconds.
Sample Correct Answer:

Becky throws a ball into the air. The height of the ball in feet, $f(t)$, after $t$ seconds can be modeled by the function shown.

$$f(t) = -16t^2 + 40t + 6$$

What is the height of the ball, in feet, 2 seconds after Becky throws it?

22

Explanation of Correct Answer:

Evaluate the function where $t = 2$. Calculate the value of $f(2)$ as shown.

$$f(2) = -16(2)^2 + 40(2) + 6$$
$$= -64 + 80 + 6$$
$$= 22$$
Question 35

Reporting Category: Modeling & Problem Solving

Common Core Standard: F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

A function is shown.

\[ y = 8x^2 + 48x - 56 \]

Over which intervals is the graph increasing, decreasing, or neither?

Above each interval on the horizontal axis, click "I" to show increasing, "D" to show decreasing, or "N" to show neither.

Scoring Rubric:

1 point
For this item, the response correctly
- identifies the letters.
Sample Correct Answer:

A function is shown.

\[ y = 8x^2 + 48x - 56 \]

Over which intervals is the graph increasing, decreasing, or neither?

Above each interval on the horizontal axis, click "I" to show increasing, "D" to show decreasing, or "N" to show neither.

Explanation of Correct Answer:

First, note that the function is a parabola of the form \( y = ax^2 + bx + c \). A parabola is symmetric about its vertex, which for the given function occurs at \( x = \frac{-b}{2a} = -\frac{48}{2(8)} = -3 \).

Thus, the function has a minimum or maximum at \( x = -3 \). Then note that the coefficient of the \( x^2 \) term is positive, so the parabola opens upward. This means that the vertex must be a minimum. Therefore, the function decreases to the left of the vertex at \( x = -3 \) and then increases to the right of \( x = -3 \).
Question 36

**Reporting Category:** Modeling & Problem Solving

**Common Core Standard:** F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.

A function is shown.

\[ h(x) = x^2 \]

What is the domain of \( h(x) \)?

Use the Connect Line tool or either of the Add Arrow tools to show the domain of \( h(x) \) on the number line.

**Scoring Rubric:**

1 point
For this item, the response correctly
- plots a graph showing all real numbers greater than or equal to 0.
Sample Correct Answer:

A function is shown.

\[ h(x) = \frac{1}{x^2} \]

What is the domain of \( h(x) \)?

Use the Connect Line tool or either of the Add Arrow tools to show the domain of \( h(x) \) on the number line.

Explanation of Correct Answer:

First, note that the given function is equivalent to \( h(x) = \sqrt{x} \). The square root of \( x \) is a real number for all nonnegative real numbers. Thus, the domain of the function \( h(x) = \sqrt{x} \) is all real numbers greater than or equal to 0. The graph of this domain is a ray that begins at \( x = 0 \) and extends to include all real numbers greater than 0.
Question 37

**Reporting Category:** Modeling & Problem Solving

**Common Core Standard:** F-BF.1c: Write a function that describes a relationship between two quantities. c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Currency conversions for British pounds and U.S. dollars are shown.
- 1 British pound = 1.59 U.S. dollars
- 1 U.S. dollar = 0.99 Canadian dollar

The functions $B(x) = 1.59x$ and $C(y) = .99y$ represent these conversions, where $x$ represents British pounds and $y$ represents U.S. dollars.

Create a composite function that represents a conversion from $x$ British pounds to $y$ Canadian dollars.

**Scoring Rubric:**

1 point
For this item, the response correctly
- identifies an equivalent function.
Sample Correct Answer:

Currency conversions for British pounds and U.S. dollars are shown.

- 1 British pound = 1.59 U.S. dollars
- 1 U.S. dollar = 0.99 Canadian dollar

The functions $B(x) = 1.59x$ and $C(y) = .99y$ represent these conversions, where $x$ represents British pounds and $y$ represents U.S. dollars.

Create a composite function that represents a conversion from $x$ British pounds to $y$ Canadian dollars.

\[ C(B(x)) = 1.5741x \]

Sequence of keypad clicks to enter the answer.

\[ C, (, B , (, x, \rightarrow, \rightarrow, =, 1.5741, x \]

Explanation of Correct Answer:

First, the function should convert $x$ British pounds to $B(x)$ U.S. dollars. Then the function should convert $B(x)$ U.S. dollars, to Canadian dollars. Thus, the correct composite function is $C(B(x)) = C(1.59x) = 0.99(1.59x) = 1.5741x$. Therefore, to go from $x$ British pounds to $y$ Canadian dollars, the correct equation is $C(B(x)) = 1.5741x$. 
Question 38

Reporting Category: Modeling & Problem Solving

Common Core Standard: F-LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

A scientist studies several colonies of bacteria. She records the number of cells in the colony every hour. Several tables containing the data are shown.

Click on the table or tables that represent exponential growth.

<table>
<thead>
<tr>
<th>Colony 1</th>
<th>Colony 2</th>
<th>Colony 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>Cells</td>
<td>Hours</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Colony 4</th>
<th>Colony 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>Cells</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

Scoring Rubric:

1 point
For this item, the response correctly
- identifies the tables.
Sample Correct Answer:

A scientist studies several colonies of bacteria. She records the number of cells in the colony every hour. Several tables containing the data are shown.

Click on the table or tables that represent exponential growth.

<table>
<thead>
<tr>
<th>Colony 1</th>
<th>Colony 2</th>
<th>Colony 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>Cells</td>
<td>Hours</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Colony 4</th>
<th>Colony 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>Cells</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

Explanation of Correct Answer:

Notice that for Colony 1 and Colony 3, the difference between consecutive numbers of cells is constant. Thus, these data increase linearly and do not represent exponential growth.

For Colony 5, the ratio between consecutive numbers of cells is constant, but the values are decreasing. This means that the data represent exponential decay.

For Colony 2 and Colony 4, the ratio between consecutive numbers of cells is constant, and the values are increasing. Thus, the only tables that represent exponential growth are the tables for Colony 2 and Colony 4.
Question 39

Reporting Category: Modeling & Problem Solving

Common Core Standard: F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Answer Key: B

A graph of an exponential function is shown.

Which equation represents the function shown in this graph?

A. $y = 2(3.2)^x$

*This answer is not correct. For this function, $y = 2$ when $x = 0$, but $y = 6.4$ when $x = 1$.***
B. \( y = 2(1.6)^x \)

*This answer is correct. For this function, \( y = 3.2 \) when \( x = 1 \) and \( y = 2 \) when \( x = 0 \).*

C. \( y = 3.2(2)^x \)

*This answer is not correct. For this function, \( y = 3.2 \) when \( x = 0 \).*

D. \( y = 1.6(2)^x \)

*This answer is not correct. For this function, \( y = 1.6 \) when \( x = 0 \).*
Question 40

**Reporting Category:** Modeling & Problem Solving

**Common Core Standard:** F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.

The height of a coconut falling from a tree can be represented by the function $h(t) = -16t^2 + 24$, where $h(t)$ is the height of the coconut, in feet, and $t$ is time, in seconds.

What is the initial height, in feet, of the coconut?

---

**Scoring Rubric:**

1 point
For this item, the response correctly
- identifies an equivalent value.
Sample Correct Answer:

The height of a coconut falling from a tree can be represented by the function \( h(t) = -16t^2 + 24 \), where \( h(t) \) is the height of the coconut, in feet, and \( t \) is time, in seconds.

What is the initial height, in feet, of the coconut?

24

Explanation of Correct Answer:

Use the Graphing Calculator tool. Select Graphing. If not already highlighted in blue, select Expressions (Y=). Enter the equation as \( y1 = -16x^2 + 2 \). Select Graph. Click on Zoom Out until the vertex is visible. The initial height of the coconut is 24 feet.
Question 41

**Reporting Category:** Modeling & Problem Solving

**Common Core Standard:** S-ID.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.  

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

**Answer Key:** C

Annie volunteers in the tutoring center at her school. The table shows how much time she spends in the tutoring center each week for six weeks.

<table>
<thead>
<tr>
<th>Week</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>105</td>
</tr>
<tr>
<td>4</td>
<td>95</td>
</tr>
<tr>
<td>5</td>
<td>85</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
</tr>
</tbody>
</table>

Which function best fits the data?

A. $y = -40x + 220$

*This answer is not correct. The student may have chosen the linear model that is satisfied by the data points for weeks 1 and 2.*

B. $y = -20x + \frac{1862}{3}$

*This answer is not correct. The student may have chosen the linear model thinking there was a constant rate of decrease.*
C. \( y = 195.5(0.852)^x \)

**This answer is correct.** The student found that an exponential model best fits the data by observing the dramatic decrease in the rate of change.

D. \( y = 257.9(0.737)^x \)

**This answer is not correct.** The student may have chosen the exponential model for weeks 1 and 2 rather than all of the weeks.

Use the Graphing Calculator tool. Select Regression. Enter the Week values in column \( x \). Enter the Time values in column Y1. Select Linear. The equation displayed eliminates Option A. Select Exponential. The equation displayed eliminates Option D. The question asks for the **best** function. Option B is eliminated, because the points do not show a constant rate of decrease. Option C is the correct answer.
Question 42

**Reporting Category:** Algebraic Concepts & Procedures

**Common Core Standard:** F-IF.7d: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

An equation is shown.

\[ y = \frac{8x^2}{4x^2 - 7x + 5} \]

What is the horizontal asymptote for this equation?

Drag the dashed line to the coordinate grid to graph the horizontal asymptote for this equation.

**Scoring Rubric:**

1 point

For this item, the response correctly:

- identifies a correct graph.
Sample Correct Answer:

An equation is shown.
\[ y = \frac{8x^2}{4x^2 - 7x + 5} \]

What is the horizontal asymptote for this equation?

Drag the dashed line to the coordinate grid to graph the horizontal asymptote for this equation.

Explanation of Correct Answer:

The horizontal asymptote is found by dividing the leading coefficient of the numerator by the leading coefficient of the denominator. Thus, the horizontal asymptote is \( y = \frac{8}{4} \) or \( y = 2 \).
Question 43

Reporting Category: Modeling & Problem Solving

Common Core Standard: F-LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Answer Key: B

The graphs of $f(x)$, $g(x)$, $h(x)$, and $k(x)$ are shown.

Which function changes at a constant rate per unit relative to $x$?

A. $f(x)$

This answer is not correct. The function $f(x)$ is not linear.
B. $g(x)$

*This answer is correct.* The function $g(x)$ is linear and has a constant rate of change.

C. $h(x)$

*This answer is not correct. The function $h(x)$ is not linear.*

D. $k(x)$

*This answer is not correct. The function $k(x)$ is not linear.*
Question 44

Reporting Category: Modeling & Problem Solving

Common Core Standard: S-ID.6b: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. b. Informally assess the fit of a function by plotting and analyzing residuals.

Answer Key: D

A residual plot is shown.

Does this residual plot suggest a good fit for the line used to generate the residuals? Why?

A. Yes. The residuals show a strong linear trend.

\[\text{This answer is not correct. The presence of a pattern in the residual plot suggests a poor fit.}\]

B. No. The data would be better modeled by a quadratic function.

\[\text{This answer is not correct. The presence of a pattern in the residual plot suggests a poor fit.}\]
C. Yes. There is an equal number of points above and below the x-axis.

*This answer is not correct. The presence of a pattern in the residual plot suggests a poor fit.*

D. No. The presence of a pattern in the residuals suggests a poor fit for this line.

*This answer is correct. The clear pattern in the residual plot suggests a poor fit for the line of best fit.*